Additive Algebra and new definition of what a Radical is

Be:

$$A^n = \sum_{1}^{A} (X^n - (X-1)^n) \tag{1}$$

A Sum that can rise till any integer Upper Limit A, where A^n is its result so its n-th Power.

Calling M_n the Complicate Modulus: $M_n = (X^n - (X-1)^n)$ the Summand is capable to rise any n-th Power of an Integer, follows from the Telescoping Sum property.

Put now $x = X/K$ with $K \in \mathbb{N}$ equal for example to 10^m.

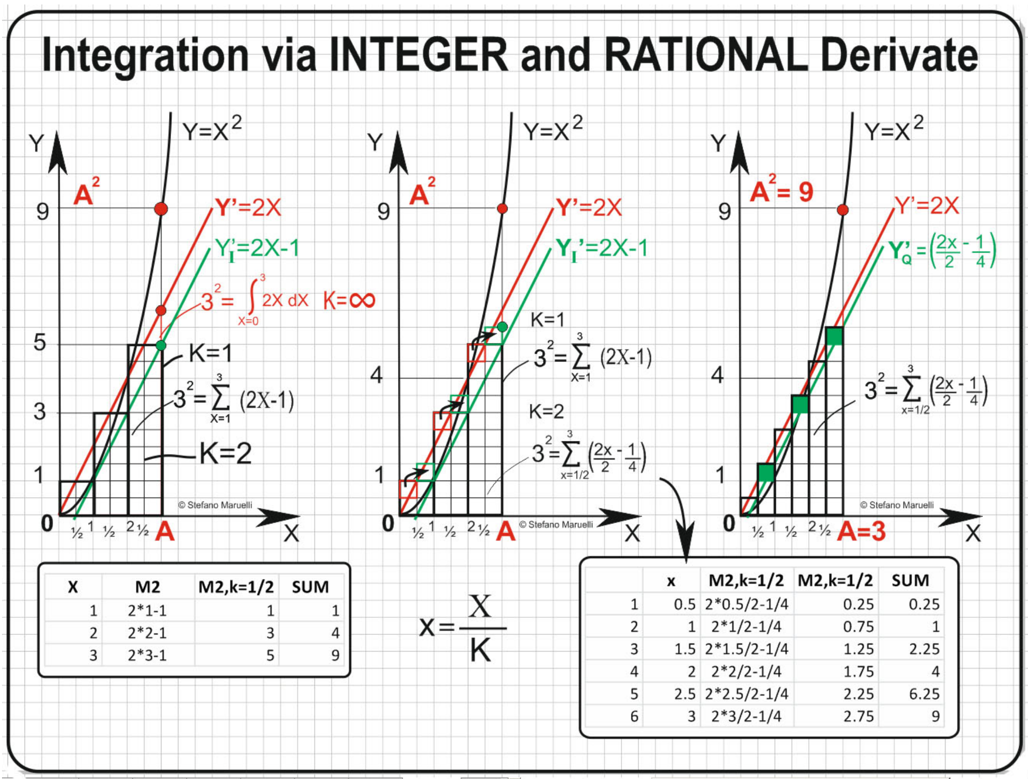
It is not hard to prove that:

$$Q^n = \sum_{1/K}^{Q} M_{n,K} \tag{2}$$

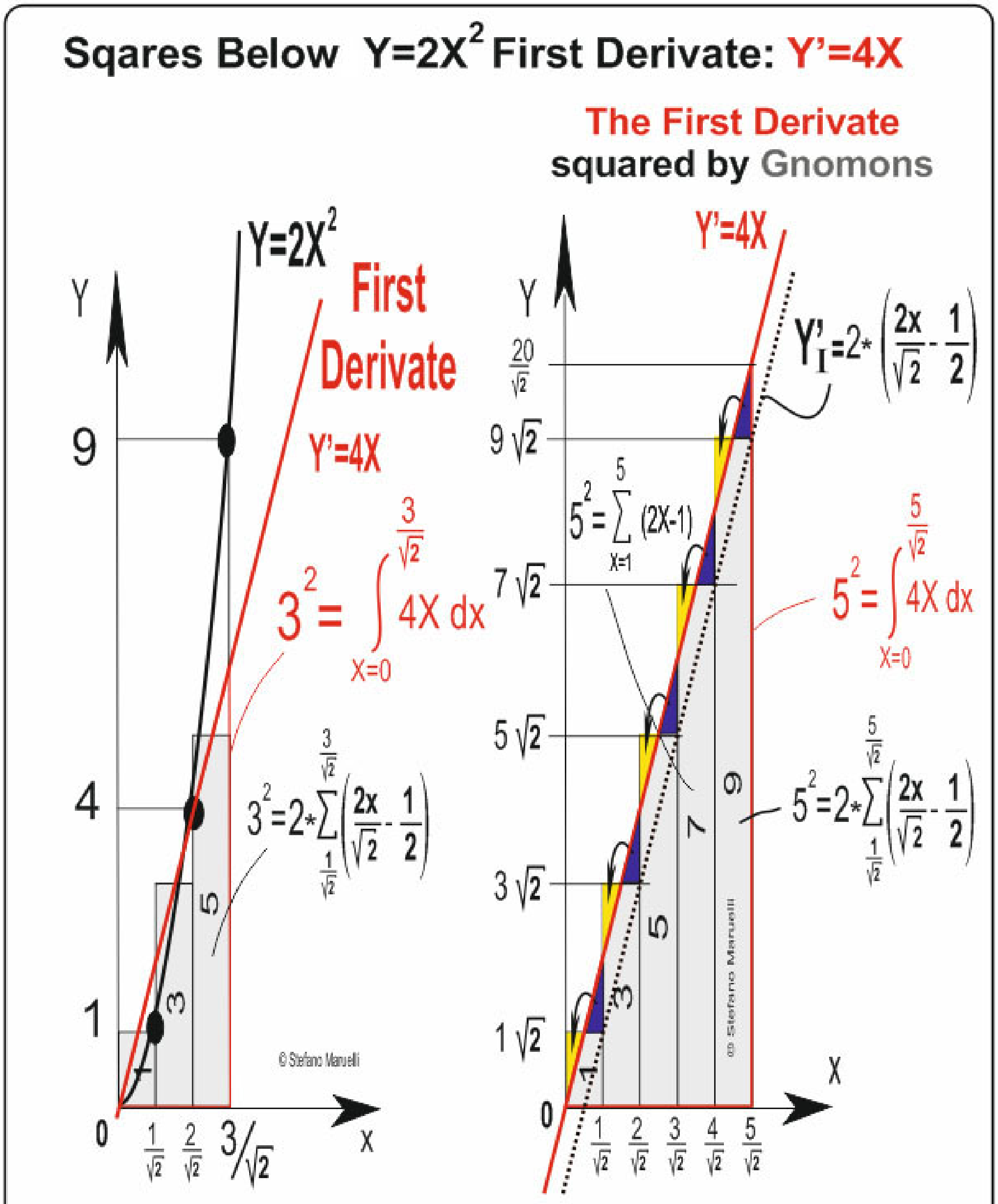
Where: Q can be an Integer but, now, also a Rational, or an Algebraic Irrational, since the New Rational Complicate Modulus becames:

$$M_{n,K} = \binom{n}{1}\frac{x^{n-1}}{K} - \binom{n}{2}\frac{x^{n-2}}{K^2} + \binom{n}{3}\frac{x^{n-3}}{K^3} - \dots + / - \frac{1}{K^n} \tag{3}$$

Here is an example of how to perform a Rational Step Sum, starting from a classic Integer Sum:



Here is an example of how to perform A Step Sum with Known Irrational Values, too:



X	x=X/sqrt(2)	Agm = Dx * Ygm =M2_k=sqrt(2):	Ygm = M2k*sqrt(2)	Sum = X^2
		Dx= x(i+1) - x(i)		
		2*M_2,K= 2*(2x/sqrt(2) -1/2)		
1	0.707106781	1	1.414213562	1
2	1.414213562	3	4.242640687	4
3	2.121320344	5	7.071067812	9
4	2.828427125	7	9.899494937	16
5	3.535533906	9	12.72792206	25

Of course at the Limit for $K \to \infty$ this Step Sum, becomes an Integral from 0 to the Upper Limit UL, where now UL can be also an Irrational Number with no restrictions.

We can now solve Any n-th Polinomial Equation if we put any Root, one by one as Upper Limit of a Sum where instead of X^n terms into the Polynomial, we put the relative Sum:

$$x^n_{x=Rx} = \sum_{1/K}^{x=Rx} M_{n,K} \tag{4}$$

where Rx is any integer, or an Algebraic Irrational, or the closest to an Irrational Root Rx Algebraic Number we can reach Step $1/K$ with an assigned suitable K .

And of course, more in general, we can rise terms of the type: aX^n

In case we know there is an Imaginary Root (due to a positive constant), then we need to change all the signs to $M_{n,K}$ so use the Imaginary Complicate Modululs is equal to:

$$M_{n,K,i} = -M_{n,K} \tag{5}$$

I will prover hereafter, that in this way we'll skip the 10th Hilbert problem and the problem of Galois Group solvability, finding a solution to any n-th degree Polinomial Equation.

The General Solving Method for Polinomyal Equations, using CMA:

Giving for know Abel-Ruffini theorem, hereafter we will show that a General Solving Algo for finding roots of any h-th Degree Polinomyal Equations exist, but produce, in case of non Rationals and non Algebraic Irrationals Roots, an infinite loop.

It also shows that a Radical is a more general concept that the one someone assign now to the solution of the equations of the type $X^n = P$ with $P \in \mathbb{R}$.

Here a simple example on the most famous, known as unsolvable, 5th degree equation $X^5 - X + 1 = 0$

We can write, with precision $K = 10^m$:

$$X^5 - X = -1 \implies \sum_{1/K}^{Rx*} M_{5,K,i} - M_{1,K,i} = -1 \tag{6}$$

Where $Rx* = p/K$ is the Rational go most closer to out true root Rx, in function of the K we choose.
Or, defining a new Specific Complicate Modulus, for this quintic equation (that define the Polynomial Root Extractor), we can write the -exact- formula:

$$\lim_{K\to\infty} \sum_{1/K}^{Rx} M_{X^5-X,K,i} = -1 \tag{7}$$

Where Rx is of course any Real Root (or zero) coming from the Integral.

We will have for so Two type of solutions:

- Integers and Rationals with a finite number of digits comes in the Computational total precision, so $R_{x*} = Rx$,

- while the other will be an apporximation till the maximum precision we are able to raise with our computer, so with our maximum K, and $R_{x*} < Rx$.

If this concept is quite easy to be understood, what follow will be little more complicate to be uderstood if onw has not yet read my Vol.1:
In case we are able to recognize the Rational or the Irrational Part of the Root produce the infinite tail of digits, we can put it /them into the Sum Index, of the Step Sum, so into K an in this way we can solve the equation in a finite number of Step.

It is also clear that in case we are not able to recognize the Rational or the Irrational with the infinite tail of digits, we have to stop somwhere the computation of the digist, just knwoing that the presence of a Rest assure us there are more digits over so our Result $RX*$ represent the first m digits of a truncated Root Rx.
For more info about the Complicate Modulus pls read my Vol.1 (link at the end of this paper).

In the quintic example, since the root is an Irrational, keeping $K = 10^m = 10^4$ we get back exactly 4 correct digits (pls note that each digit you get rising m of 1, will be an exact decimal, one, not an approximated one).
The solution obey to this proven rule, is , theoretically due to the Algebraic construction of the solution, is capable also to work till the limit for $K \to \infty$ with an infinite number of digit, so it is not an approximation, that is just a technical problem as the same happens when you've as result f.ex. $\sqrt{2}$ and you've to show the value of the numerical result so someone. So a new more general $\sqrt{}$ sign can be forged.

Returning to our example, so to the Quintic: the Modulus we must use to find the Roots in the Recursive Difference from the known constant, here: -1 looking to have $Rest = 0$ (or as close to zero as we can in case of Irrational Roots) is:

$$M_{x^5-x,K,i} = -5 * \frac{x^4}{K} + 10 * \frac{x^3}{K^2} - 10 * \frac{x^2}{K^3} + 5 * \frac{x}{K^4} - \frac{2}{K^5} \tag{8}$$

That differs from the 5th line of the Tartaglia's triangle $M_{5,K,i}$ (represent X^5), for the last term, just, since we have to add with the right sign $M_{1,K,i}$ (represent X), that is the constant $\frac{1}{K}$ summed K times.

And still if the Number R_x* represent the Closest Rational to an Irrational Solution comes from a long computation of very long Rational digit numbers, we can affirm that we can always find any R_x Root, using what I hope is clear now is a more General Root (Algebraic) Extractor (GRE theorem).

So the point is: I've produced a new more general definition for a Radical Root, finding a General Polinomial Root Extractor that can works on any polinomial and the classic n-th Root algo that is capable to work (in general) just on Hypercube, is just a sub class of this General Polinomial Root Extractor.

Moreover, any Ring has the same properties shown here for Polinomials, will lead to similar zeros, or Roots. But this will require an Abstract Algebra discussion that is not what I would like to do here, since my purphose is to introduce young students to a new point of view or the known classic algebra.

solution of x5-x+1=0.jpg

Solution for X^5-X+1=0			r1= -1.1673039782614		r1*=1.1673	K=10000	
		The Sum this part rise to r1^5			this rise to r1		Constant
X	x	5x^4-10x^3+10X^2-5X+1	Sum		1/K	difference:	-1
1	0,0001	1E-25	1E-25		0,0001		-1,0001
2	0,0002	3E-19	3E-19		0,0001		-1,0002
3	0,0003	2,1E-18	2,4E-18		0,0001		-1,0003
4	0,0004	7,8E-18	1,02E-17		0,0001		-1,0004
5	0,0005	2,1E-17	3,12E-17		0,0001		-1,0005
6	0,0006	4,65E-17	7,77E-17		0,0001		-1,0006
7	0,0007	9,03E-17	1,68E-16		0,0001		-1,0007
11645	1,1645	0,000919292	2,141398353		0,0001		-0,023101647
11646	1,1646	0,000919608	2,142317961		0,0001		-0,022282039
11647	1,1647	0,000919924	2,143237884		0,0001		-0,021462116
11648	1,1648	0,00092024	2,144158124		0,0001		-0,020641876
11649	1,1649	0,000920556	2,145078679		0,0001		-0,019821321
11650	1,165	0,000920872	2,145999551		0,0001		-0,019000449
11651	1,1651	0,000921188	2,146920739		0,0001		-0,018179261
11652	1,1652	0,000921504	2,147842244		0,0001		-0,017357756
11653	1,1653	0,000921821	2,148764064		0,0001		-0,016535936
11654	1,1654	0,000922137	2,149686201		0,0001		-0,015713799
11655	1,1655	0,000922454	2,150608655		0,0001		-0,014891345
11656	1,1656	0,00092277	2,151531425		0,0001		-0,014068575
11657	1,1657	0,000923087	2,152454513		0,0001		-0,013245487
11658	1,1658	0,000923404	2,153377916		0,0001		-0,012422084
11659	1,1659	0,000923721	2,154301637		0,0001		-0,011598363
11660	1,166	0,000924038	2,155225675		0,0001		-0,010774325
11661	1,1661	0,000924355	2,15615003		0,0001		-0,00994997
11662	1,1662	0,000924672	2,157074702		0,0001		-0,009125298
11663	1,1663	0,000924989	2,157999691		0,0001		-0,008300309
11664	1,1664	0,000925306	2,158924997		0,0001		-0,007475003
11665	1,1665	0,000925624	2,159850621		0,0001		-0,006649379
11666	1,1666	0,000925941	2,160776562		0,0001		-0,005823438
11667	1,1667	0,000926259	2,161702821		0,0001		-0,004997179
11668	1,1668	0,000926576	2,162629398		0,0001		-0,004170602
11669	1,1669	0,000926894	2,163556292		0,0001		-0,003343708
11670	1,167	0,000927212	2,164483504		0,0001		-0,002516496
11671	1,1671	0,00092753	2,165411033		0,0001		-0,001688967
11672	1,1672	0,000927848	2,166338881		0,0001		-0,000861119
11673	1,1673	0,000928166	2,167267047		0,0001		-3,29532E-05
11674	1,1674	0,000928484	2,168195531		0,0001		0,000795531

This is not a Numerical Solution, since it is an Algebraic Root Extractor, as the extraction of a known n-th Root is.

In both case, in fact, the Root will produce an Irrational Number require a truncation to print the Radical, or thet it was left under the Root Sign.

So I hope I've proved here that the concept of Radical and Root is much wider and lead to new interesting simple results can be presented without any abstract definition of Groups / Rings / Ideals etc...

Reference: Maruelli The two hand clock Vol.1 -

`https://www.researchgate.net/publication/351495686_Maruelli-The-Two-Hand-Clock-Vol1-Revi`
`fullTextFileContent`